Students THINK: A Framework for Improving Problem Solving

The THINK interaction framework (see **fig. 1**) was used in classrooms with preservice teacher interns at three grade levels: first, second, and sixth. Students who used THINK demonstrated greater growth in problem solving than students who did not use the framework. Students' growth was measured by having all students solve four word problems with explanations *before* implementing THINK and four word problems with explanations *after* the students in half the classrooms had used THINK. Individual interviews were used to obtain students' explanations of each problem solution. The problem solutions and interview transcripts were scored using a rubric based on Pólya's problem-solving process (see **fig. 2**). Students earned a holistic score for each problem, and the four problem scores were combined into a total score. The percentage of change between pretest and posttest scores was greater for the students in the THINK classrooms than for those in the regular classrooms (see **table 1**). In fact, the THINK students' problem-solving skills were enhanced to such a degree that the school's principal and teachers have chosen to implement the framework schoolwide.

Research has shown that students who communicate and reason about mathematics problems have a greater potential for understanding the concepts that underlie the problems (Ball and Bass 2000; Cobb et al. 1997; Schoenfeld 1985; Stein, Grover, and Henningsen 1996). The ability to think about, and reflect on, the problem-solving process is an important component of learning mathematics. Communicating these ideas with classmates to cooperatively solve a mathematics problem is another step in understanding mathematics. THINK was developed to help children learn to share their reasoning and thinking related to problem solving. Another important outcome for children is to "learn to question and probe one another's thinking" (NCTM 2000, p. 63). THINK provides a structure to support listening to and examining others' ideas.

The THINK framework was developed for a Title I urban elementary school in which 31 percent of the student population came from an economically disadvantaged background. The state had required the school to initiate an improvement plan for mathematics instruction; this mandate provided the impetus for the research. The mathematics goal of the school improvement plan stated that "stu-

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dents will acquire skills to improve mathematical problem solving." More specifically, the action plan to meet the goal included students' use of a systematic approach to solving problems and use of metacognition to explain their mathematical processes and thinking.

To satisfy the action plan, the school administration and staff selected a commercial problem-solving series, *The Problem Solver* (Goodnow and Hoogeboom 1987), as a supplement to the regular textbook series. We formed a professional development school (PDS) partnership between the school and the university and undertook a collaborative effort to explore components of problem-solving instruction and the development of students' problem-solving skills. Our research team was composed of the school principal, teachers, and preservice teacher interns, along with the author, a mathematics educator.

THINK Plan

During preliminary work sessions, the genuine interest of the school principal and faculty in helping students develop mathematical reasoning and problem-solving skills was readily apparent to the author and the teacher interns. The teachers were invested in seeking the most effective instructional models for addressing these concerns.

Two instructional methods have been identified in research literature as means to enhance mathematical reasoning. One focuses on metacognitive training, in which students monitor their own thinking while using appropriate information and strategies during problem solving (Carr and Biddlecomb 1998; Desoete, Roeyers, and Buysse 2001; Mevarech 1999; Mevarech and Kramarski 1997). A second method focuses on cooperative learning (e.g., Slavin 1996) and the importance of sharing mathematical reasoning with others as a community of learners (Ball and Bass 2000; Cobb et al. 1997; Hiebert et al. 1997). Kramarski and Mevarech (2003) further confirmed the effectiveness of these two instructional strategies when they examined the effects on mathematical reasoning of combining the two methods. Instructional strategies that combined cooperative learning and metacognitive training in eighth-grade classrooms resulted in higher student problem-solving achievement than in classrooms using cooperative learning or metacognitive training alone (Kramarski and Mevarech 2003). The THINK interaction framework was used to train students in metacognitive thinking and to guide students' discourse while they solved problems cooperatively.

THINK Classrooms Compared with Regular Classrooms

The teaching team consisted of five teacher interns; two in first grade, two in second grade, and one in sixth grade, as well as one sixth-grade teacher (hereafter, these are all referred to as teachers). The teachers randomly selected one class from each grade level to participate in problem solving using the THINK framework (hereafter referred to as THINK classrooms) and one class from each grade level to participate in all problem-solving exercises without using the THINK framework (hereafter referred to as regular classrooms). The instructional sequence lasted for five weeks, and the students in all classrooms were involved in the same grade-appropriate problem-solving activities every day, with the exception of the THINK classroom students using the THINK framework. Students in all classrooms solved the same problems.

A typical week in the regular classrooms began with teachers' presenting a problem to the class, sharing ways the problem might be solved, and looking for a solution to the problem. Following two days of whole-class work on problems, students spent two days in small groups solving problems. The last day of each week was devoted to individual problem solving, which enabled students to apply the skills developed during the whole-class and small-group work.

The THINK classrooms followed a format identical to that used in the regular classrooms except that the teachers elicited students' thinking and participation through questions related to the THINK framework. They discussed each problem, beginning with "T—Talk." They described the situation and explained what the problem asked as well as identified important information. The discussion progressed to focus on "H—How"—how the problem could be solved. Beyond brainstorming ideas for solving the problem, students were asked to justify and explain why they thought their plan

Figure 1

THINK interaction framework

TALK about the problem with one another. Describe the situation. Explain what the problem is asking. Talk about the important information.

HOW can the problem be solved? Have each person share ideas for how to solve the problem. Ask others how and why their plan will work.

IDENTIFY a strategy for solving the problem. Use it. Talk about how to use your strategy. Is your strategy working, or do you need to choose another one to solve the problem?

NOTICE how your strategy helped you solve the problem. Have each person share how the strategy helped him or her understand and solve the problem.

KEEP thinking about the problem. Does your answer make sense? If you can think of another way to solve the problem, share it.

Table 1

Percentage of Growth in Problem Solving

	First Grade	Second Grade	Sixth Grade
THINK classes	24	14	26
Regular classes	14	9	13

Figure 2

Problem-solving scoring rubric

	4 points	3 points	2 points	1 point
Understand	Response indicates insight and complete understanding of the problem	Response indicates understanding of the problem	Response indicates partial understanding of the problem	Response indicates misunderstanding of the problem
Plan	Makes original/ creative plan to solve the problem Organizes data concisely and with insight Uses one or more strategies to solve problem	Shows workable plan to solve the problem Organizes data appropriately Chooses a strategy to solve the problem	Shows a plan that will not solve the problem Partially organizes data Chooses an inappropriate strategy to solve the problem	Produces unworkable plan Does not organize data Chooses no strategy or chooses an incorrect strategy
Solve	Shows clear, well-organized implementation of the plan Clearly shows logical processes used in implementation Uses data that fit the information given in the problem	Shows correct implementation of the plan Shows some evidence of the processes used Makes few or no errors in data	Shows partially correct implementation of the plan Shows little evidence of the processes used Produces work having many errors	Shows incorrect implementation of the plan Produces work that is unrelated to the problem
Check	Attains clear, reasonable solution that is meaningful to the problem Clearly labels all parts Gives clear, insightful reasons to explain the accuracy of the solution If solution is not reasonable, shows evidence of choosing another strategy	Finds reasonable, acceptable solution Labels most parts Gives an explanation for the solution If solution is not reasonable, shows some evidence of redoing the problem	Produces partially acceptable solution Labels no parts or few parts Gives incomplete or unclear explanation for the solution If solution is not reasonable, shows no evidence of redoing the problem	Attains unreasonable solution or a solution that is unrelated to the problem Uses no labels Gives no explanation for the solution
Score:	Understand _____ Plan _____ Solve _____ Check _____ = Total _____			

would work. Next, in the "I—Identify" phase, each group identified a strategy or plan for solving the problem. An essential aspect of this phase was to encourage students to think about and evaluate the effectiveness of the selected plan while it was being executed. This approach helped students reason about the mathematics and the thinking they were using, thus resulting in a greater awareness of their power to navigate the problem-solving process. To add to the sense of mathematics power, we asked the students to "N—Notice" how the strategies they used helped them solve the problem. Finally, we reminded students that they were expected to "K—Keep thinking" about the problem and determine whether the solution made sense. The class then shared other possible ways to solve the problem.

During the two days allotted for small-group problem solving, the students in the THINK classrooms used the THINK framework to guide their discussions. We expected each student to make a contribution to the discussion during each phase of THINK. To that end, we gave the students color-coded sticks representing each step of the framework; after sharing for a particular phase, the contributing student placed a stick of that phase's color in a pile. The sticks helped students monitor their contributions as they learned to use the THINK framework. The teachers reported that after the first two weeks, using the sticks was no longer necessary. As one teacher commented, "I started to notice the students were sharing their thinking for each of the THINK phases without using the sticks to keep track."

On the last day of the week the individual problem solving in a THINK classroom was identical to that in the regular classrooms. However, teachers

Figure 3

Problem-solving interview questions

- What does the problem want you to find?
- What important information did the problem give you?
- What strategy did you use to solve the problem? Or what did you do to solve the problem?
- How would you tell someone else to solve the problem?
- Does your answer make sense? Why or why not?

noticed that THINK students were able to capitalize on their prior work of solving problems in small groups during the week. Participating with others in a structured interaction about problems gave the students experiences to draw on to solve problems on their own.

THINK Results

To determine growth in problem-solving skills, students solved four problems before and after the five weeks of instruction as well as responded to five interview questions (see **fig. 3**) for each problem. We evaluated students' problem-solving skills using a rubric that measured each student's ability to understand, to plan, to solve, and to check each problem (see **fig. 2**). The data from 112 student pretest–posttest scores and 112 student interviews demonstrate that across grade levels the students in the classrooms using the THINK framework showed greater overall growth in problem-solving abilities (21 percent growth), as measured by the change in pretest and posttest scores on four problems, than did the students in the classrooms working in groups to solve and discuss problems without the framework as a guide (12 percent growth). The data at each grade level confirm that the percentage of growth in problem-solving scores was higher for the students using the THINK framework than for students not using the framework (see **table 1**).

Further analysis of the scores using analysis of covariance, with the pretest scores as the covariate, indicates that the effect of the THINK framework was significant: $F(1, 111) = 7.495$, $p < .01$. By using the pretest scores as a covariate, this analysis controls for differences in the groups before the unit and suggests that those students who used THINK during problem solving outperformed students who did not.

The teacher conducting the interview recorded the student responses during the interview, and we later compiled the responses by grade level and classified them into one of two groups: the THINK classroom group versus the regular classroom group. We analyzed the responses for complexity of response, mathematical accuracy, mathematical reasoning, and the use of mathematical language. The qualitative examination of the students' responses to interview questions about each problem provides additional evidence of growth in problem-solving skills, particularly regarding students' ability to communicate and reason about the mathematics problems. Overall, the students in the THINK classrooms responded to the interview questions in greater depth while using more appropriate mathematical language than did their counterparts in the regular classrooms. A sample of individual student interview responses for each grade level (**figs. 4, 5,** and **6**) illustrates the typical pattern of responses. At each of the three grade levels, a representative student response from a THINK classroom and one from a regular classroom characterize students' thinking about the problems before and after the five weeks of instruction.

The more complex responses from the THINK students showed greater mathematical reasoning and communication skills than did the responses from the regular classroom students. For example, first-grade students in both the THINK and regular classrooms often provided vague reasoning and little mathematical language in response to questions about the problems before the five weeks of instruction, as is evident in the responses in **figure 4**. Such responses as "I don't know" and "To color it" lend little insight into a student's thinking.

In her interview responses following the unit (see **fig. 4**), the student in the THINK classroom was able to use mathematical language to communicate her thinking about a problem that requires the solver to extend a pattern. She continually used the word *pattern* and was able to communicate the process she used to determine the solution, saying, "I thought 'shell, shell, clam, crab, shell, shell, clam, crab, shell, shell, . . . ' When I got to the end, I needed the next one, so I went back and saw that clam was after shell. I saw that two times in a row, so I knew clam was next." She reasoned that the picture of the pattern was important to her ability to solve the problem, saying, "We had the picture of the shells to show us. If it [the picture] wasn't there, I would say, 'Wait a minute, what?' Trying to make a pattern. That's important."

Figure 4

Two first-grade students' interview responses

Problem solved before the unit

Four fish line up to use the diving board. The red fish is first in line. The blue fish is in front of the yellow fish. The green fish is in front of the blue fish. What color is the last fish in the line? (Goodnow and Hoogeboom 1987, Grade 1, T49)

Before Five-Week Unit	THINK Class Student's Solution	Regular Class Student's Solution
1. What does the problem want you to find?	I'm not sure.	Find the color and color it.
2. What important information did the problem give you?	I can't remember.	To get the right color.
3. What strategy did you use (what did you do) to solve the problem?	I thought yellow and green went together and blue and green went together.	To color it.
4. How would you tell someone else to solve the problem?	I don't know.	To color it.
5. Does your answer make sense? Why or why not?	I don't know.	Yes, but I'm not sure why.

Problem solved after the unit

One day Connie saw a trail of shells in the sand. There were snail shells, crab shells, and clam shells in the trail. Connie followed the trail until it went behind a big rock. Connie saw a pattern in the trail, so she knew what shell was behind the rock. Look for the pattern that Connie saw. What shell will Connie find behind the rock? [The problem showed a picture of a series of objects in this order: shell, shell, clam, crab, shell, shell, clam, crab, shell, shell, then a rock hiding the next type of shell.] (Goodnow and Hoogeboom 1987, Grade 1, P8)

After Five-Week Unit	THINK Class Student's Solution	Regular Class Student's Solution
1. What does the problem want you to find?	The pattern, what goes next after the last shell. Draw the shell to show the answer.	The thing that goes after it.
2. What important information did the problem give you?	We had the picture of the shells to show us. If it (the picture) wasn't there, I would say, "Wait a minute, what?" Trying to make a pattern. That's important.	The words tell what goes next.
3. What strategy did you use (what did you do) to solve the problem?	I thought, "Shell, shell, clam, crab, shell, shell, clam, crab, shell, shell, . . . " When I got to the end, I needed the next one, so I went back and saw that clam was after shell. I saw that two times in a row, so I knew clam was next.	I'm not sure what strategy I used. I looked at the shell to see what comes next.
4. How would you tell someone else to solve the problem?	I would tell them to look for the pattern. If they still couldn't figure it out, I would show them that clam goes after these two shells and these two shells. Then I would ask them, "What goes after these two shells?"	Look at what is before the rock.
5. Does your answer make sense? Why or why not?	Yes; after I did this, I double-checked it by saying the pattern again and again. It was the same, so I knew it was right.	Yes, because there is a shell.

Figure 5

Two second-grade students' interview responses

Problem solved before the unit

Lacy Ladybug is at the bottom of the plant, but her babies are at the top. Lacy has to crawl past 20 bugs on the plant to reach the top. She crawls past 14 bugs. Whoosh! A wind blows her down past 7 bugs. Lacy turns around and starts to crawl up the plant again. She crawls past 10 bugs. Where is Lacy now? (Goodnow and Hoogeboom 1987, Grade 2, P5)

Before Five-Week Unit	THINK Class Student's Solution	Regular Class Student's Solution
1. What does the problem want you to find?	Where is the ladybug.	Where she is.
2. What important information did the problem give you?	How many times she crawled up the tree.	It tells you where she is going to be.
3. What strategy did you use (what did you do) to solve the problem?	I counted.	I did what the problem said to do.
4. How would you tell someone else to solve the problem?	Count how many times she went up.	I would tell them to read the problem.
5. Does your answer make sense? Why or why not?	Yes; I don't know why.	Yes; I don't know why.

Problem solved after the unit

Secret Agent Buzzy looked carefully at the paper. "What is happening to the letters that were on this paper?" he wondered. When Buzzy found the paper at nine o'clock, there were 26 letters of the alphabet on it. After one hour there were only 22 letters on the paper. After two hours there were only 18 letters on the paper. After three hours there were only 14 letters left. Letters were disappearing every hour! How many letters would be left after five hours? (Goodnow and Hoogeboom 1987, Grade 2, P14)

After Five-Week Unit	THINK Class Student's Solution	Regular Class Student's Solution
1. What does the problem want you to find?	How many letters would be left on the paper after 5 hours.	How many letters are on the paper 5 hours later.
2. What important information did the problem give you?	How many letters were left after the 4th hour and that 4 letters disappeared every hour.	Letters were disappearing every hour.
3. What strategy did you use (what did you do) to solve the problem?	I counted backwards by 4s. Took away, 14 – 4 is 10.	I figured out in my head how many letters were on the paper every hour.
4. How would you tell someone else to solve the problem?	I'd tell them to count backward 4 from 14 to see how many letters are left.	I don't know.
5. Does your answer make sense? Why or why not?	No, because I found how many letters would be left after 4 hours, and I should have found how many letters would be left after 5 more hours. So I should have counted back by 4s from 14, five times	Yes; I don't know.

In contrast, the responses of the student in the regular classroom were less complex. She did not use the word *pattern* as she explained what the problem wanted her to find, responding, "The thing that goes next." To identify the important information given in the problem, she wrote, "The words tell what goes next"; to describe the strategy she used, she replied, "I'm not sure what strategy I used. I looked at the shell to see what comes next"; and to describe how she would tell someone else to solve the problem, she responded, "Look at what is before the rock." Although both students correctly solved the problem, the THINK classroom student was able to communicate her reasoning and thinking in greater depth than the regular classroom student.

After the unit, students in the THINK classrooms were more likely to find errors in their reasoning and calculations and to correct their incorrect solutions than were their peers in the regular classrooms. The second-grade student from the THINK classroom (see **fig. 5**) had initially produced an incorrect solution to the problem. He explained how he solved the problem: "I counted backwards by 4s. Took away, 14 – 4 is 10." Through the process of communicating with the interviewer, the student reasoned that his solution did not make sense and was able to communicate how he would change his solution. In response to the question "Does your answer make sense?" he said, "No, because I found how many letters would be left after 4 hours, and I should have found how many letters would be left after 5 more hours. So I should have counted back by 4s from 14, five times." The student in the regular classroom also generated a correct solution; however, her responses were vague and she used little mathematical language to reason or communicate her thinking. In response to the question "Does your answer make sense and why?" she said, "Yes; I don't know." When asked what strategy she used to solve the problem, she responded, "I figured out in my head how many letters were on the paper every hour." Although this response does indicate that she solved the problem mentally, it does not articulate what she did to "figure" in her head. In response to the same question, the student from the THINK classroom said, "I counted backwards by 4s."

Similarly, after the unit the students at the sixth-grade level in THINK classrooms regularly used mathematical reasoning and language to express their thinking about problems. These students also recognized errors and made corrections more often than students in the regular classrooms. **Figure 6** shows students' responses that illustrate the overall difference in the two groups. The THINK student's responses were more detailed, and she was able to explain the processes she used to solve the problem and how she would recommend that another student solve the problem, saying, "Make a graph to keep track of how many pizzas were sold for every 6 cheese, multiply the other kinds by 5, and add the numbers together." The student from the regular classroom not only responded with vague explanations, such as "Read it, and find the problem and solve it," but also incorrectly solved the problem and did not reason mathematically about the appropriateness of his solution.

THINK Again

The results of this research and work in classrooms highlight several benefits of using a framework such as THINK as a metacognitive training tool to guide students' interactions as they work cooperatively to solve problems. Working with a group of peers to solve mathematics problems gives students an opportunity to develop problem-solving skills through discourse. Structuring group discourse using a metacognitive tool such as the THINK framework enhances students' mathematical reasoning behaviors. Participating in cooperative-group problem solving with an emphasis on discourse about mathematical reasoning improves students' ability to recognize important aspects of problems, identify effective solution strategies, and justify the solution using mathematical language. Students in the THINK classrooms enhanced their abilities to internalize and subsequently use the problem-solving processes.

A future direction for research and classroom use of the interaction framework might be to include an individual precursor to the group conversation to stimulate individual students' thinking about problems. The framework could be modified to "I—THINK" by introducing "I—independent thinking" by individual students before they contribute to the group communication about problems. Adding this element might strengthen individual students' ability to think independently about how to solve problems they encounter.

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Figure 6

Two sixth-grade students' interview responses

Problem solved before the unit
Saul is helping his brother, Jason, do carpentry and general repair jobs. Jason gave Saul some money for supplies. Saul bought some tools for $20.80 and then spent one-half of what was left for wood. He spent one-third of what was left after that on paint and another third on buckets and brushes. He returned with $13.20. How much money did Jason give Saul? (Goodnow and Hoogeboom 1987, Grade 6, P11)

Before Five-Week Unit	THINK Class Student's Solution	Regular Class Student's Solution
1. What does the problem want you to find?	How much money one person gave another.	How much money did Jason give Sal.
2. What important information did the problem give you?	Spent $20.80, 1/2 of what was left, 2/3 after that, returned with $13.20.	To subtract, but I don't know what else.
3. What strategy did you use (what did you do) to solve the problem?	I subtracted from $100.	I added and got $37.60.
4. How would you tell someone else to solve the problem?	Subtract from $100.	Work it out.
5. Does your answer make sense? Why or why not?	Yes, but I'm not sure why.	No; I don't know why.

Problem solved after the unit
The Square Table Pizza Palace sells cheese, plain, sausage, and mushroom pizzas. Marty found one Saturday night that for every 6 orders of cheese pizza, there were 10 orders of plain pizza, 7 orders of sausage pizza, and 4 orders for mushroom pizza. If 30 cheese pizzas were ordered that night, how many pizzas did they sell altogether? (Goodnow and Hoogeboom 1987, Grade 6, P3)

After Five-Week Unit	THINK Class Student's Solution	Regular Class Student's Solution
1. What does the problem want you to find?	The number of pizzas sold in the night.	How many pizzas were sold altogether.
2. What important information did the problem give you?	For every 6 cheese pizzas sold, there are 7 sausage, 4 mushroom, and 10 plain pizzas sold. The number of cheese pizzas sold was 30.	How many pizzas there were.
3. What strategy did you use (what did you do) to solve the problem?	I made a graph. Because there were 5 groups of 6 cheese pizzas in 30, I multiplied the other kinds of pizza by 5 and added them all together.	I added up all the numbers and got 57. Then I divided it by 5, and the answer was 11 R2
4. How would you tell someone else to solve the problem?	Make a graph to keep track of how many pizzas were sold for every 6 cheese, multiply the other kinds by 5, and add the numbers together.	Read it, and find the problem and solve it.
5. Does your answer make sense? Why or why not?	Yes, because $6 \times 5 = 30$, and so you know to multiply all the other kinds by 5.	Yes; I don't know.

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